

Active layer model for wall-bounded turbulence

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The active-layer model for wall-bounded turbulence hypothesizes that the non-linear terms are large only in a thin layer near the wall, and hence the turbulence in the region outside the active inner layer can be modeled as a linear fluctuating flow driven by the active layer. This hypothesis is tested using data obtained from a direct simulation of turbulent channel flow. It is found that the nonlinear effects are the strongest near the wall with a maximum at around $y^+ = 20$ and, outside the near-wall region, these involve primarily the cascading mechanism leading to dissipation.

1. Introduction

Laboratory experiments and numerical simulation have shown that the turbulent activity in wall-bounded turbulence is the highest in the immediate neighborhood of the wall, in the viscous and buffer layers. Therefore, nonlinear effects may be expected to be strongest in this region. A possible model for the turbulent field might therefore be to consider the turbulence in the region outside an active inner layer as a linear fluctuating flow driven by the active layer (Fig. 1).

2. Analysis

We subdivide the flow field in a parallel mean flow, $U(y)\delta_{i1}$, and a fluctuating field, u, v, w, p . By elimination of the pressure from the momentum equations, making use of the continuity equations in the process, the following equation for the v -component is found (Landahl, 1967):

$$L_{OS}(v) = q \quad (1)$$

where L_{OS} is the (space-time) Orr-Sommerfeld operator,

$$L_{OS}(v) = (\partial/\partial t + U\partial/\partial x)\nabla^2 v - U''\partial v/\partial x - \nu\nabla^4 v \quad (2)$$

and q contains all the nonlinear terms,

$$q = \nabla^2 T_2 - \partial^2 T_i/\partial x_i \partial x_2 \quad (3)$$

where

$$T_i = \partial\tau_{ij}/\partial x_j \quad (4)$$

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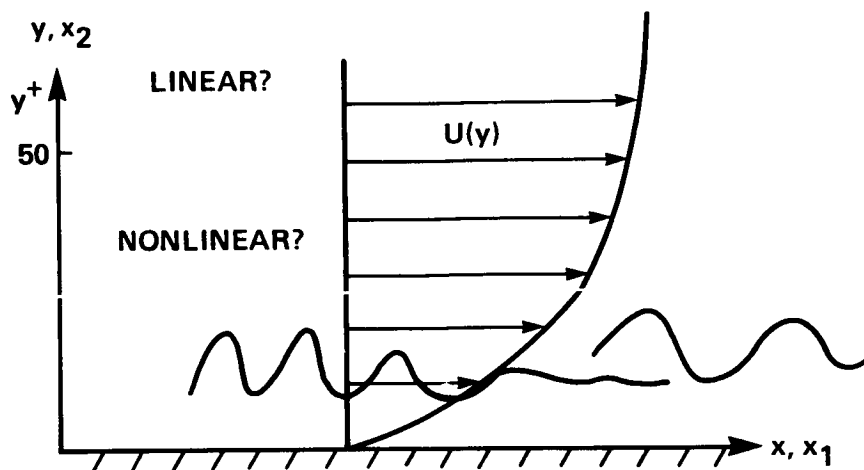


FIGURE 1. A model of wall-bounded turbulence consist of active non-linear inner layer and a linear fluctuating outer layer driven by the active layer.

with

$$\tau_{ij} = -u_i u_j \quad (5)$$

The solution of (1) assuming q to be given may be determined by applying Fourier transformation in x , z and t , which yields

$$(U - c)(\hat{v}'' - k^2 \hat{v}) - U'' \hat{v} - (1/ik_x)(d^2/dy^2 - k^2)^2 \hat{v} = \hat{q}/ik_x \quad (6)$$

where the caret denotes a Fourier-transformed quantity, k_x and k_z , denote the x - and z -components of the wave number k , respectively, and $c = \omega/k_x$.

3. Calculation of the Reynolds stress

The basic hypotheses in the active-layer model are that the nonlinear terms are large only in a thin region near the wall and that the flow in the region outside this layer may be found by considering q as a given and driving an outer linear fluctuating flow field. From the statistics of q one may then determine any desired statistical quantity such as the mean Reynolds shear stress,

$$\tau = -\rho \langle uv \rangle \quad (7)$$

By applying Fourier transform to the component equations and using Parseval's formula relating quadratic mean quantities to their transforms, one finds, upon neglecting viscous terms, that the wave number spectrum, S_τ , for the Reynolds shear stress, is related to the spectrum for the v -fluctuations through

$$S'_\tau = (k_z/k)^2 (d/dy)(U' S_v^c) + (k_x/k)^2 U'' S_v^c \quad (8)$$

where S_v^c denotes the spectrum of v in a reference frame convected with the mean velocity $U(y)$, and the prime denotes differentiation with respect to y . From (6), it

follows that for small viscosity the v -fluctuations may receive their largest contributions from the region near $k_x = 0$, in which case the second term in (8) will be negligible, and with $k = k_z$ one finds after integration that

$$S_\tau = U' S_v^c \quad (9)$$

a result that may be shown to be consistent with Prandtl's mixing-length hypothesis.

4. Application to channel flow turbulence

The basic hypothesis on which the present model is based, namely, that the nonlinearities are important only in a thin region near the wall, may be tested with the aid of the numerical turbulence simulations. A fairly extensive data base for turbulent channel flow has been generated in the NASA-Ames numerical simulations, although limited to fairly low Reynolds numbers. From the computed data, the nonlinear terms of (1) may be extracted and appropriate statistical quantities determined. Since the detailed temporal evolution is not so easily accessible because of the way the data are stored, for a preliminary assessment of the soundness of the basic hypothesis of the present model, the spatial mean and power spectrum of a single time realization was determined from the computed velocity field. The results are shown in Figs. 2 and 3. In Fig. 2, the root-mean-square value of q is presented (arbitrary scale). It has a maximum at around $y/h = 0.1$ (h = channel half width), which corresponds to $y^+ = 18$, and drops off to a value of about one tenth of the maximum towards the center of the channel. The spectra shown in Fig. 3 for different distances from the wall are found to be fairly flat in the wall and buffer regions, with the cutoff in k_z at a higher value than for k_x , reflecting the dominance of high streamwise elongation of the dominating structures. The spectrum at $y^+ = 76$ (Figs. 3g and 3h), however, which is well outside the buffer layer, has its main contribution from the high wave-number range (both in k_x and k_z). This probably reflects the nonlinear cascading mechanism involved in dissipation. The production of Reynolds stress resulting from this spectrum is not large, however, since it receives its main contributions from the low k_x -end. The part of the q -spectrum responsible for the production can therefore be expected to be even more concentrated in the near-wall region.

5. Conclusion

The model proposed for wall-bounded turbulence, namely, that the nonlinear driving of the turbulent fluctuations is concentrated in the near-wall region, in the viscous and buffer regions, has been tentatively examined using data from the NASA-Ames channel-flow simulations. The preliminary findings are that the nonlinear effects are the strongest near the wall, with a maximum at around $y^+ = 20$, and that outside the near-wall region they involve primarily the cascading mechanism leading to dissipation. The mean and the spectra were obtained from a single time realization; for a more general treatment, one would need to work with ensemble averages over a large number of realizations, as well as to employ spectra in

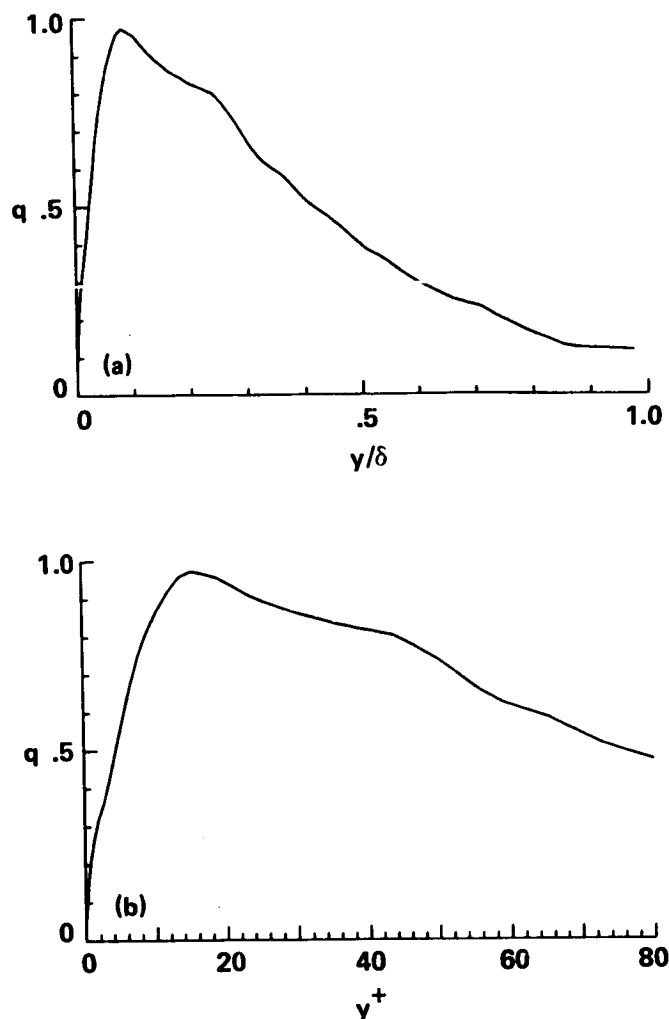
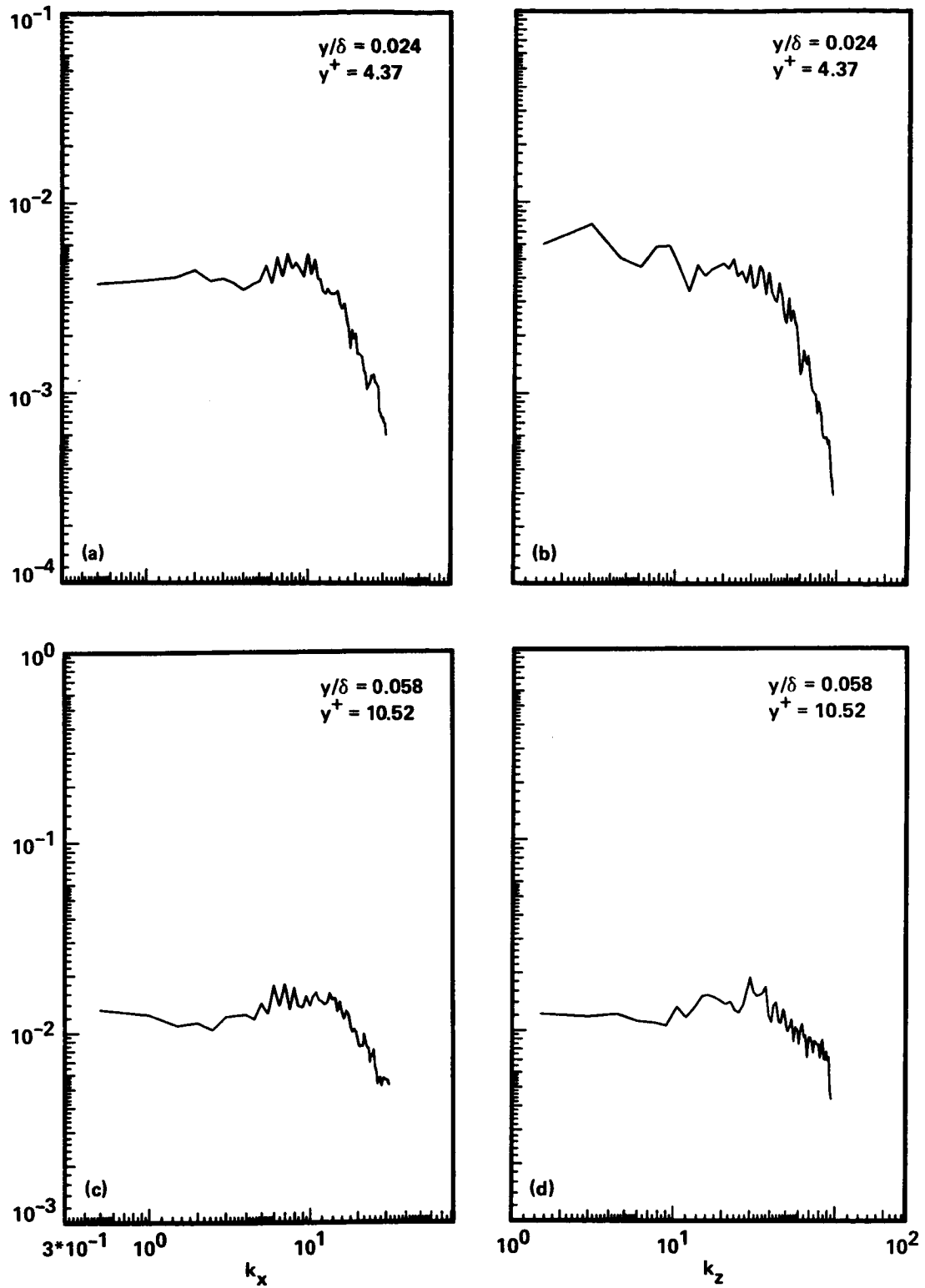


FIGURE 2. Profiles of the root-mean-square values of q in Eqn. (1).

a frame of reference convected with the local mean velocity. If found to be sound, this model will lead to a reasonably simple procedure for determining the Reynolds stresses and other statistical quantities through a comparatively simple linear calculation making use of a universal model for the nonlinear processes in the near-wall region, the statistics of which may be found from numerical simulations carried out at modest Reynolds numbers.

REFERENCES

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FIGURE 3. Spectra of q at different y -locations.

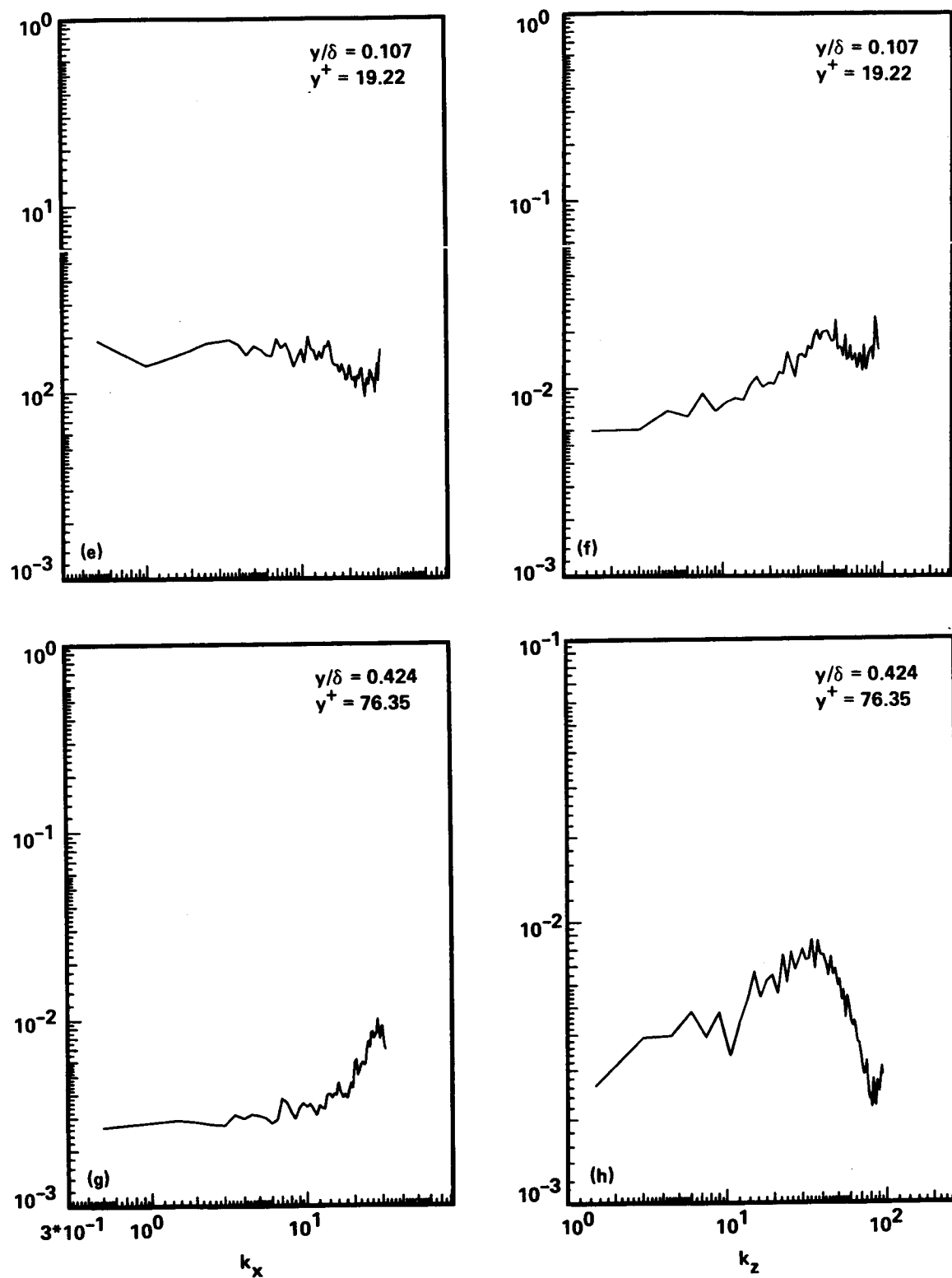


FIGURE 3. ... continued.